Table 1: Results from the Gaussian and uniform 25% thinning examples.

Thinning and weighting	Integrated sidelobe level (ISL)	Peak sidelobe level (SL)	Beamwidth	Weight dynamic range
Full array (Fig. 1)	-9.84 dB	-13.5 dB	1.04°	0 dB
Gaussian, unweighted (Fig. 2)	-6.06 dB	-15.7 dB	1.26°	0 dB
Gaussian, min SL (Fig. 3)	-7.48 dB	-23.7 dB	1.43°	15.3 dB
Gaussian, min SL (Fig. 4)	-8.28 dB	-24.6 dB	1.54°	26.7 dB
Gaussian, min ISL (Fig. 5)	-8.00 dB	-17.2 dB	1.43°	8.3 dB
Gaussian, min ISL (Fig. 6)	-9.78 dB	-18.7 dB	1.54°	12.3 dB
Uniform, unweighted	-2.71 dB	-16.7 dB	0.98°	0 dB
Uniform, min SL	-1.95 dB	-19.4 dB	1.09°	29.9 dB
Uniform, min ISL	-3.15 dB	-16.0 dB	1.09°	7.6 dB

[3] P. K. Weber, R. M. Schmidt. B. D. Tylkowski, J. Steck, “Optimization of random sparse 2-D transducer arrays for 3-D electronic beam steering and focusing,” Proc. 1994 IEEE Ultrasonics Symp., Cannes, France.

[4] J.O. Erstad, S. Holm, “An approach to the design of sparse array systems,” Proc. 1994 IEEE Ultrasonics Symp., Cannes, France.

[5] S. Holm, B. Elgetun, “Optimization of the beampattern of 2D sparse arrays by weighting,” Proc. 1995 IEEE Ultrasonics Symp., Seattle, Washington.

[6] V. Murino, A. Trucco, and C. S. Regazzoni, “Synthesis of unequally spaced arrays by simulated annealing,” IEEE Trans. Sign. Proc., vol. 44, No. 1, pp 119-123, Jan. 1996.

[7] S. Holm, B. Elgetun, “Properties of the beampattern of sparse arrays,” submitted for publication, May 1996.

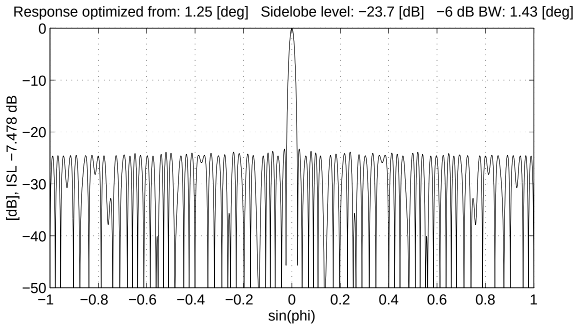

Fig. 3 Beam pattern of thinned array with weighting for minimum peak sidelobe level

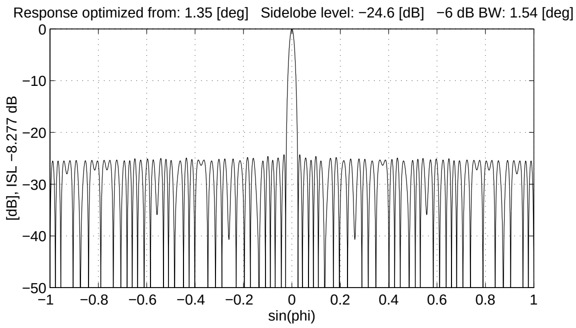

Fig. 4 Beam pattern of thinned array with weighting for minimum peak sidelobe level

sidelobe performance than the full array.

It is of interest also to study the weightings found in the two cases. The weighting for minimum integrated sidelobe energy is smoother, tapers then ends down, and has less dynamic range than the peak sidelobe weighting. One can say that this weighting alters the original beampattern much less (Fig. 2 compared to Fig. 5 and Fig. 6) and therefore less is required from the weighting also.

Another example was made where the thinning was performed with a uniform distribution. The series of removed elements was: {6,10,15,23,31,32,36,37,39,41,42,52,53,55,57,63}. This thinning removes more elements around the center of the array than the previous one. The thinned array therefore has inferior properties and is more difficult to optimize. In this case the unweighted array is near optimum with respect to integrated sidelobes, and performance comparable to the full array is not possible to achieve even when the beamwidth is increased. For the same beamwidth, this example shows a decrease of 1.2 dB in integrated sidelobe ratio when it is optimized for.

Table I summarizes the parameters of both examples.

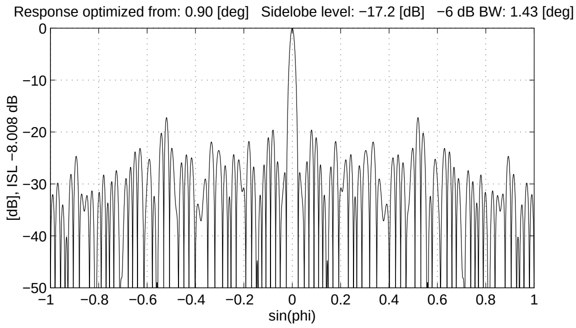

Fig. 5 Beam pattern of thinned array with weighting for minimum integrated sidelobe energy

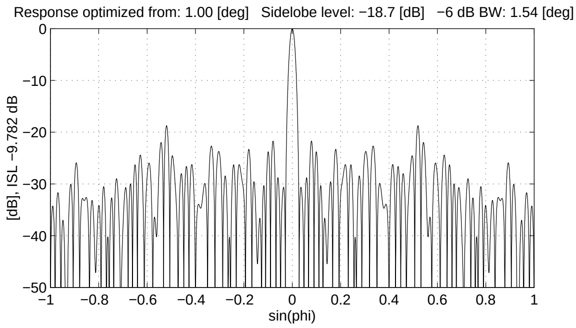

Fig. 6 Beam pattern of thinned array with weighting for minimum integrated sidelobe energy

5. CONCLUSION

In most cases minimization of the peak sidelobe is not worth the cost of increased integrated sidelobe level. Minimization of the integrated sidelobe level results in control of both sidelobe energy and sidelobe level as opposed to a minimization of sidelobe level alone. It is therefore recommened as the criterion to use for sparse array optimization.

REFERENCES

[1] J. W. Adams, “A new optimal window,” IEEE Trans. Sign. Proc., vol. 39, No. 8., pp 1753-1769, Aug. 1991.

[2] R. E. Davidsen, S. W. Smith, “Sparse geometries for two-dimensional array transducers in volumetric imaging,” Proc. 1993 IEEE Symp. Ultrasonics, Baltimore, USA.

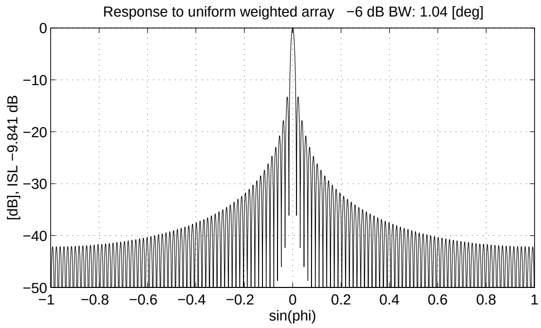

Fig. 1 Beam pattern of unweighted full array

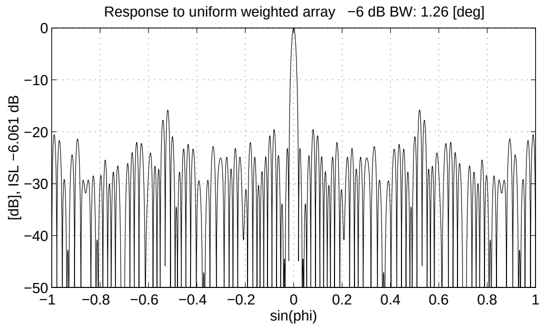

Fig. 2 Beam pattern of unweighted thinned array (Gaussian thinning)

therefore often the most relevant image quality criterion: Minimize the integrated sidelobe energy, under the condition that the peak sidelobe does not exceed a prescribed limit. In [1] this method was developed and examples are given of window designs that have the same peak sidelobe level as the Hamming window, while reducing the integrated sidelobe energy by 5 dB. Alternatively, a window with the same integrated sidelobe energy, but with 4.6 dB lower peak sidelobe may be found.

3. SPARSE ARRAYS

The need for 3D volumetric data acquisition has created a need for 2D arrays in medical ultrasound imaging. The hardware requirements of such systems are formidable as the number of channels in the system may increase from 48-128 to several thousands. This has triggered an interest in sparse arrays, where elements are missing in the aperture given by (1). Thinned apertures also create the need to find optimal sparse array weightings or layouts.

Some of the work on sparse arrays has been done by manual optimization to a flat sidelobe pedestal [2], others have used the minimum peak sidelobe criterion ([4], [5], [6]). The only exception is in [3] where the somewhat heuristic criterion of finding the best approximation to the full, unweighted array's beampattern is used. The minimization of the integrated sidelobe level has as far as we know never been used on sparse arrays.

4. EXAMPLES

We have developed optimization methods based on linear programming [5] and on quadratic programming, in order to compare. This has been tested on an example consisting of a 128 element array with $\lambda/2$ element spacing. The beampattern in the wave-number domain is shown in Fig. 1. The advantage of using the wave-number domain rather than giving the result as a function of angle, is that the beampattern will be equivalent to the Fourier transform in time-frequency signal processing. The full array is then thinned so that 25% of the elements are removed symmetrically according to a Gaussian thinning (more likelihood for thinning near the ends). The thinned elements in the first half of the array (elements 1-64) are: {3,6,7,8,10,11,12,14,15,16,18,21,22,30,34,41}.

In Figs. 2-6 a comparison is made. The ratio of energy in the sidelobes and the mainlobe is also output. It is found by integrating the squared beampattern in the two regions and finding the ratio. The border is defined as twice the -6dB beamwidth on each side. Figs. 3 and 5 have the same beamwidth and should be compared, and Figs. 4 and 6.

The main results of a comparison are:

- The integrated sidelobe level (ISL) decreases by 1.4-2.2 dB as the array is optimized for minimum peak sidelobe level at the expense of 14-22% increase in beamwidth (Fig. 2 compared to Fig. 3 and Fig. 4).
- Further reduction of integrated sidelobe ratio can be achieved when it is optimized for. Minimization of the ISL results in a decrease of 1.9-3.7 dB compared to the unweighted array (Fig. 2 compared to Fig. 5 and Fig. 6)
- The cost of getting the few large sidelobe peaks down by 5-6 dB is an overall increase in sidelobe level that results in an increase in 0.5-1.5 dB in integrated sidelobe level. (Fig. 3 compared to Fig. 5 and Fig. 4 compared to Fig. 6).
- It is possible to obtain comparable integrated sidelobe ratio for a thinned array and a full array, compare Fig. 1 to Fig. 6. The beamwidth has however increased by 48%, while the peak sidelobe is about 5 dB lower. Thus the weight-optimized thinned array has equivalent or better

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Minimum Sidelobe Energy versus Minimum Peak Sidelobe Level for Sparse Array Optimization

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ABSTRACT

Both minimization of the peak sidelobe level and minimization of the integrated sidelobe energy may be used as physically justifiable criteria of optimality in window design in signal processing, and in spatial taper design in array processing. In this contribution, it is shown that the integrated sidelobe criterion often is more relevant of the two, when the weights on the elements of sparse arrays are to be optimized.

1. INTRODUCTION

In window design in digital signal processing there are two frequency domain criteria of optimality:

1. Minimum peak sidelobe level resulting in Dolph-Chebyshev weighting.
2. Minimum integrated sidelobe energy resulting in the prolate-spheroidal window which is approximated by the Kaiser-Bessel window.

In spectral analysis the first criterion minimizes the effect of spectral leakage from discrete frequency components. The second criterion is related to the estimation of a low spectral level in a background of broad-band noise at the other frequencies. This situation is not so common in spectral estimation.

In imaging systems such as medical ultrasound systems, the window is used as a spatial taper over an array. The beampattern is equivalent to the Fourier transform of the window. The minimization of the maximum sidelobe is a criterion which is related to imaging of a strong reflecting point target in a non-reflecting background containing other point targets. A typical scenario is imaging of point targets in water. Although this is not a clinically relevant imaging scenario, it is typical for testing of imaging systems. However in certain organs of the human body, the imaging scenario may approximate this situation. This applies for instance to imaging of valve leaflets inside the fluid-filled cardiac ventricles.

The alternative criterion of minimization of the integrated sidelobe energy is directly related to image contrast when imaging a non-reflecting area like a cyst or a ventricle in a background of reflecting tissue. As such it is found much more often in the human body than the previous scenario.

2. BACKGROUND

Assume a linear array with equidistant elements spaced $d = \lambda/2$ apart (Nyquist sampling). The beam-pattern is:

$$W(u) = \sum_{n=1}^{N} w_n e^{j2\frac{\pi}{\lambda}nud} = \sum_{n=1}^{N} w_n e^{j\pi nu} \qquad (1)$$

where the argument is $u = \sin\varphi$ and φ is the angle of incidence on the array. The minimization of the peak sidelobe level, δ_s, is equivalent to the minimax criterion and can be stated as a linear programming problem:

$$\begin{gathered} W(0) = 1 \\ \text{Min } \delta_s \\ |W(u)| \le \delta_s \qquad \forall u \in [u_1, 1] \end{gathered} \qquad (2)$$

where u_1 is the cut-off, i.e. sine of the start-angle of the sidelobe region.

The minimization of the integrated sidelobe level, σ_s, can be stated as a quadratic optimization problem:

$$\begin{gathered} W(0) = 1 \\ \text{Min } \sigma_s = \text{Min } 2\int_{u_1}^{1} |W(u)|^2 du \\ |W(u)| \le \delta_s \qquad \forall u \in [u_1, 1] \end{gathered} \qquad (3)$$

An additional constraint to keep the sidelobe level below a threshold value has been added. Depending on the input value for δ_s, this constraint may or may not have any effect on the outcome of the optimization.

The last criterion described above captures the features of both the optimization criteria. It is