ACOUSTIC FIELD SIMULATION FOR ARBITRARILY SHAPED TRANSDUCERS IN A STRATIFIED MEDIUM

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Abstract

An efficient approach to the simulation of the acoustic field from an ultrasound transducer is the impulse response method. One of its disadvantages is its inflexibility with respect to source geometry. Another limitation is the assumption of a homogeneous medium. This is a direct consequence of the method since it is based on finding the diffraction impulse response from the source surface to the field point. When the medium contains aberrations this can no longer be done. In this case one has to use a direct summation of the Rayleigh integral, since in principle each path from a point on the source to the observation point has a unique time delay and attenuation profile. Traditionally this has not been considered feasible due to processing time limitations. Using modern work stations, many problems of interest are now feasible to solve with reasonable processing time using a simulator based on direct summation.

The method including the delay and amplitude calculations through the layered medium is described. The simulator has been used to find the resulting field after propagation through a layered medium such as a transducer with a dome or a transducer viewing through a layer of fat. Examples of such simulations are given and compared with measurements.

Introduction

For sources in a rigid baffle the velocity potential is given by the Rayleigh integral:

$$\Phi(r,t) = \int_S\int \frac{u_n(r_0, t - r/c)}{2\pi r} dS \qquad (1)$$

An ultrasound transducer array element has an angular response which lies somewhere between that of a source in a pressure release baffle and that of a source in a rigid baffle ([4], p.171). Thus the above solution is not exact. The discrepancies are evident at large incidence angles. On-axis the Rayleigh approximation gives good results, and presently most simulation tools are based on it.

The impulse response method is based on two assumptions: i) The source is plane, i.e. the lateral dimensions and the radius of curvature are large compared to the wavelength [2]. ii) The source vibrates in a single mode, i.e. the surface velocity is separable in a time-dependent and a space-dependent part. By defining the diffraction impulse as

$$h(r,t) = \int_S\int \frac{w(r_0)\delta(t - r/c - t_0)}{2\pi r} dS \qquad (2)$$

where w is the apodization. The velocity potential can be expressed as a convolution [1]:

$$\Phi(r,t) = u(t) \otimes h(r,t) \qquad (3)$$

Thus by using properties of the geometry of the radiator, the problem is reduced to a 1-dimensional integral and the dimensionality of the field calculation is reduced.

One of the underlying assumptions of the impulse response method is that the path from the radiator to the summation point is independent on actual position, i.e. the medium is homogeneous. Thus this method cannot be used when the field is to be found in an aberrating medium. In this case one has to give up the speed advantage and solve the Rayleigh integral directly taking the medium properties into account for each path from source to field point. This solution is derived here starting with the wave equation.

Theory

The basic equations

The derivation of the wave equation is based on two basic principles: i) Mass is neither created nor destroyed. ii) The rate of change of momentum of a portion of the medium equals the force applied to it (Newton's second law). The equations can be found in [5]. In the law of balance of momentum a nonviscous medium is assumed.

A third equation models the relationship between mass density and pressure. An equation after Stokes,

$$p = \frac{1}{K}(\frac{\rho - \rho_0}{\rho_0}) + R\frac{\partial}{\partial t}(\frac{\rho - \rho_0}{\rho_0}), \qquad (4)$$

is useful for this purpose ([6] p220). The equation models how efficiently the pressure field compresses the material. The first term models the normalized density change to be proportional to the pressure p with the proportionality constant being the compressibility K. The second term is a correction term to correct for sound absorption. When the mass is compressed or decompressed, energy will transform to heat. This sound absorption is modelled to be proportional to the rate of change of the normalized mass density. The proportionality constant is R. R may be a function of frequency.

Let us assume a homogeneous medium. By assuming $u << c$ (ok. [7] p229) and neglecting gravity (ok. [5] p.9) we get:

$$\nabla^2\Phi(\underline{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi(\underline{r},t) = -RK\frac{\partial}{\partial t}\nabla^2\Phi(\underline{r},t), \qquad (5)$$

where $c^{-2} = \rho_0 K$ and

$$\underline{u} = -\nabla\Phi(\underline{r},t) \quad and \quad p = \rho_0\frac{\partial}{\partial t}\Phi(\underline{r},t) \qquad (6)$$

Eq. 5 is a wave equation with absorption. The wave equation is linear so if a continous wave is a solution, then a sum of continous waves is also a solution. From Fourier analysis we know that it is possible to make all kind of pulses from continuous waves, so assuming continuous waves is not a limitation. Assuming continuous waves, eq. 5 takes the form:

$$\nabla^2\Phi(\underline{r}) + k^2\Phi(\underline{r}) = 0, \quad k^2 = \frac{\omega^2}{c^2}\frac{1}{1-iRK\omega} \qquad (7)$$

This equation is equal to Helmholtz equation, but k is now complex. Assuming $(RK\omega)^2 << 1$, it is possible to use $k^2 = {k_r}^2 + i{k_c}^2$ where $k_r = \frac{\omega}{c}$ and $k_c = i\sqrt{RK\omega}k_r$.

Solution by the ray-tracing approximation

The Helmholtz equation without absorption is solved in several books [5] by using the ray-tracing approximation. We use the same method to solve Helmholtz equation with absorption included. Assume:

$$\Phi(\underline{r}) = A(\underline{r})e^{ik_{r0}S(\underline{r})} \qquad (8)$$

where $k_{r0}S(\underline{r})$ is the phase of the wave, $k_{r0} = \frac{\omega}{c_0}$ and c_0 is the reference velocity. $S(\underline{r})$ is the acoustical distance. The acoustical distance is the geometrical distance multiplied by the refraction index $n = \frac{c_0}{c}$. By inserting eq. 8 into eq. 7 and separating the real and imaginary terms we get two equations. The real terms give

$$(\nabla S)^2 = n^2 + \frac{\nabla^2 A}{{k_{r0}}^2 A} \qquad (9)$$

and the complex terms give

$$2\nabla A\nabla S + A\nabla^2 S = -RKA\omega k_{r0}n^2 \qquad (10)$$

The ray-tracing approximation is found by assuming $\frac{\nabla^2 A}{{k_{r0}}^2 A} << n^2$. Using this approximation eq. 9 is reduced to the eikonal equation, i.e. an equation for the phase. Eq. 10 is the amplitude equation with absorption included. When S and A is found the solution of the wave equation with absorption is

$$\Phi(\underline{r},t) = A(\underline{r})e^{ik_{r0}S(\underline{r})}e^{-i\omega t} \qquad (11)$$

The ray path

On interfaces between two different materials the refraction index, $n(\underline{r})$, will be discontinuous. This produces refraction and reflection. It can be shown from the eikonal equation that the reflected and refracted rays lie in the same plane as the incident ray and the normal to the interface [8]. The angle of refraction in this plane satifies Snell's law.

The amplitude of a ray

The equation for the amplitude is given in eq. 10. Using $\underline{I} = \frac{\omega^2}{2\rho c}A^2\underline{s}$ (ok if $|\frac{\nabla A}{A}| << k_{r0}n$) eq. 10 can for a homogeneous medium be rewritten as:

$$\nabla(\underline{I}) = -\mu I \qquad where\ \mu = RK\frac{\omega^2}{c} \qquad (12)$$

Volume integration over a ray tube at both sides and using the divergence theorem on the left side give:

$$I_2 dB_2 - I_1 dB_1 = \int_V -\mu I dV \qquad (13)$$

It means that power out of a ray tube minus power in is equal to the absorption loss along the ray tube. The solution of eq. 13 is given by:

$$IdB = I_1 dB_1 e^{-\mu\sigma} \qquad (14)$$

The variable σ is geometrical distance along the ray and dB is the area of the cross section of the ray tube.

On the boundary between two different materials some of the energy is reflected and some is transmitted. By requiring continuity of the pressure and the normal velocity and also that the total amount of power entering the boundary must equal the total amount of power leaving one gets:

$$\frac{\text{Tr. ray power}}{\text{In. ray power}} = 1 - R^2 \qquad (15)$$

where

$$R = \frac{Z_o/cos\alpha_o - Z_i/cos\alpha_i}{Z_o/cos\alpha_o + Z_i/cos\alpha_i} \qquad (16)$$

$Z = \rho c$ and α is the angle between the normal of the boundary and the ray.

Combining eq. 14 and 15 for a stratified medium with n boundaries give a relation between the intensity at the start and the end of the ray path:

$$\frac{I_n}{I_0} = \frac{dB_0}{dB_n} \exp^{-\sum_{i=0}^{n} \mu_i \sigma_i} \prod_{i=0}^{n-1} (1 - R^2{}_{i(i+1)}) \qquad (17)$$

The attenuation is divided in three terms. The first, dB_0/dB_n, is due to broadning of the ray. For a homogeneous medium this is $1/r^2$, i.e. a spherical wave. The second term represents absorption due to transformation of energy into heat. The last term represents reflection losses. If $|\frac{\nabla A}{A}| << k_{r0} n$ the amplitude of the pressure is given by $\sqrt{I \rho_0 c}$ and the amplitude of the particle velocity is given by $\sqrt{I/\rho_0 c}$.

Simulation method

The presented simulator consists of four parts: 1) A transducer geometry module which samples an arbitrarily shaped source, 2) a medium module which computes delays and attenuation through a stratified medium, 3) an observation module which specifies the two- or three-dimensional surface where the field is to be found, and 4) a window-based user interface module where parameters can be set.

In the described simulator the method is based on the Rayleigh integral of eq. 1 modified in the following ways:

- The surface of the source is divided into points separated by a distance less than half the wavelength.
- We are tracing ray paths from the observation points to all points on the transducer. This is done by iteration and using Snell's law.
- The phase term is found from the distance along the path and the velocities of sound in each layer.
- The square root of eq. 17 replaces the factor $1/r$ in the Rayleigh integral and is used to find the attenuation caused by diffraction, reflection, and absorption along each path.

It would have been desirable that the input is the electrical input, as done in [3], where the Mason model for the transducer is used in combination with a field simulator. The transfer function from voltage and current excitation to pressure and velocity is given by the model, and then the output is used as an input to the field simulator.

In our present simulator we do not excite with the electrical pulse. Instead we excite with a pulse that resembles as much as possible the pressure pulse measured at the focal point on the acoustical axis. At this point one will get coherent summation of the Rayleigh integral. This means that we excite with a measured approximation to the normal component of the surface velocity in eq. 1.

The frequencies in the pulse is distributed around the center frequency. The absorption factor depend on the frequency. So far we use only the center frequency to calculate absorption along a ray path.

The attenuation factor due to the broadning of the ray, dB_0/dB_n, is found for each ray calculating two neighbour rays. Each with the same start point but with small changes in the ray vector. The two neighbour rays span out a ray-tube. The area of the cross section of the ray tube, dB, can then be calculated. dB_0 is the area of the cross section close to the start of the ray and dB_n is the area of the cross section after travelling trough n layers.

Simulations and Experiments

The simulation program has been used for quantifying the effect of time delay quantization in beamforming [9], and for finding the effect of medium-generated phase aberrations [10].

The simulations and measurements presented in this paper are based on an annular transducer with radius of curvature 50 mm and diameter 15 mm. The transmitted pulse has 5 oscillations and center frequency at 5 MHz.

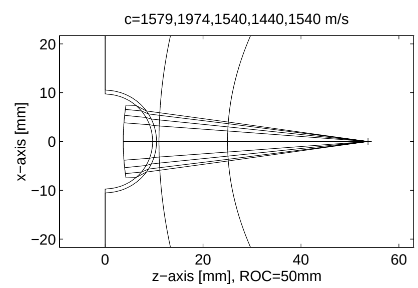

Figure 1: Raytracing through a layer of fat and a dome

The simulations can be done in two or three dimensions. An example with ray-tracing through a layer of fat and a dome is presented using 2D simulations. The media and some ray-paths are presented in fig. 1. It is possible to see refraction of the rays in the dome.

Fig. 2 shows the calculated phase aberrations in the transducers focal point,50 mm, and at a depth of 35mm. As a reference the phase aberrations for the homogeneous case is also calculated. At 50 mm the phase aberrations for the homogeneous case is of course zero for all elements on the transducer. When the dome and a layer of fat are included, we get phase aberrations at 50 mm. The outer ring shows phase aberrations of about 25 % of a wave period. At the depth of 35 mm we get less phase aberrations by including the dome and a layer of fat compared to the homogeneous case.

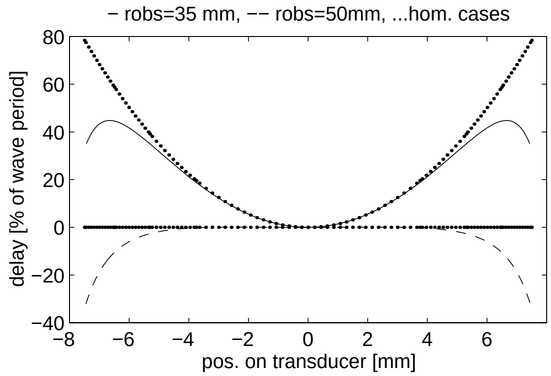

Figure 2: Phase aberrations through a layer of fat and a dome compared with the homogeneous case

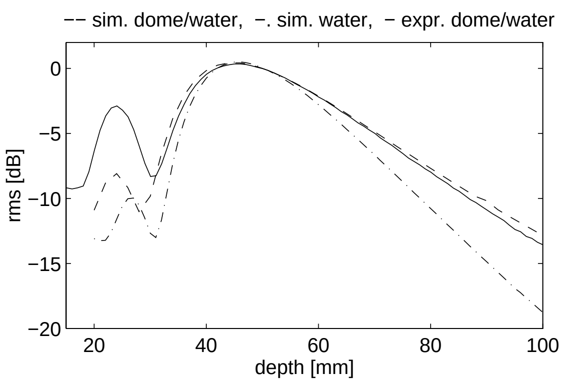

Figure 3: On-axis intensity in water for a 5 MHz annular transducer

Examples of 3D simulations and measurements are shown in fig. 3 and 4. The simulations are performed with and without dome for a 5 MHz annular transducer immersed in water. The experimental curves are obtained from measurements of the pressure field excited by a 5 MHz transducer with a dome. The measurements were obtained using a hydrophone with diameter 0.4 mm. We see that by including the dome in the simulation model we get a better fit with experiments both on-axis and off-axis.

Conclusion

The presented simulation method for stratified media make better predictions to measurements than simulations assuming a homogeneous media.

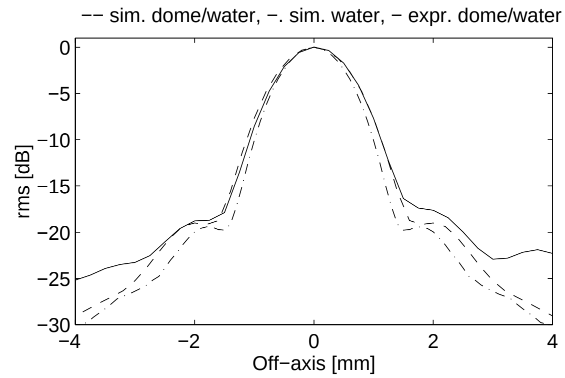

Figure 4: Off-axis intensity in water for a 5 MHz annular transducer. Observation at focal depth; 50 mm

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